HW 9: NUMBER THEORY

1. Prove that for no integer $n > 1$ does n divide $2^n - 1$.
2. Determine all functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying

$$f(x^3 + y^3 + z^3) = f(x)^3 + f(y)^3 + f(z)^3$$

for all $x, y, z \in \mathbb{Z}$.

3. Find all functions $f : \mathbb{N} \to \mathbb{N}$ satisfying

$$f(n) + 2f(f(n)) = 3n + 5,$$

for all $n \in \mathbb{N}$.

4. Show that no positive integers x, y, z can satisfy the equation

$$x^2 + 10y^2 = 3z^2.$$

5. Let n be an integer greater than 2. Prove that $n(n-1)^4 + 1$ is the product of two integers greater than 1.

6. Let p be a prime number. Prove that there are infinitely many multiples of p whose last ten digits are all distinct.